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DETERMINATION OF THE EQUIVALENT DAMPING CONSTANT OF A SERIES ASSEMBLY OF ELASTIC SPRINGS

BY

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Abstract. In this paper a dynamical system composed by a solid mass attached at two elastic springs bounded in a series assembly is studied. In a certain hypothesis one obtains the equivalent damping constant of the assembly in a free vibrations regime (mechanical vibrations).

Key words: elastic spring, damping constant.

1. Introduction

The purpose of this paper is the finding of damping constant for an elastic system of two springs bounded in a series assembly. We present the condition where this equivalent constant has an analogous formula by that of equivalent elastic constant.

One considers two helicoidally elastic springs with the elastic constants K_1, K_2 and damping constant C_1, respectively C_2. The springs are bounded in series assembly in vertical position (Fig. 1) with the mass (m_1) between springs and the mass (m_2) attached at the extremity of the second spring.

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One denotes l_1, l_2 the current lengths of the springs and l_{10}, respectively l_{20} the lengths of the springs in undistorted state. One denotes m_1 and m_2 the masses of the system. For the second spring one considers the case of subcritical damping:

$$C_2^2 < 4K_2m_2 . \tag{1}$$

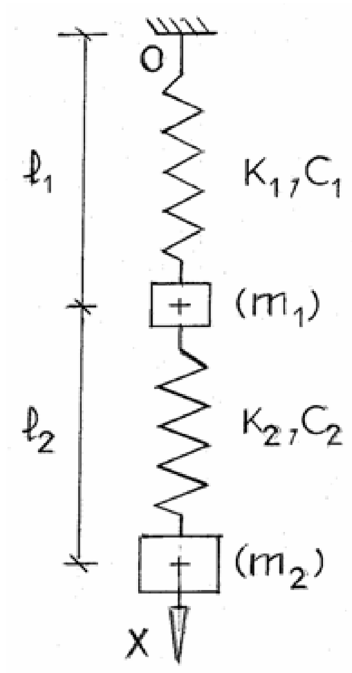

Fig. 1 − Series assembly of elastic springs.

2. Contents

If the system is taken out from the static equilibrium position, the dynamical equation for the mass (2) is:

$$m_2\ddot{x}_2 = m_2g + F_{e_2} + F_{a_2} , \tag{2}$$

where x_2 is the coordinate, g − the gravitational acceleration, F_{e_2} − the elastic force and F_{a_2} − the damping force of the second spring.

The elastic force is given by Hooke's relation (Buzdugan *et al*., 1979; Teodorescu, 1984; Teodorescu, 1988; Teodorescu, 1997):

$$F_{e_2} = -K_2(l_2 - l_{20}) \,. \tag{3}$$

The damping force is given by (Mangeron & Irimiciuc, 1978; Mangeron & Irimiciuc, 1980; Mangeron & Irimiciuc, 1981; Buzdugan *et al.*, 1979):

$$F_{a_2} = -C_2(\dot{x}_2 - \dot{x}_1) \,, \tag{4}$$

where x_1 is the coordinate of mass (1) and $l_2 = x_2 - x_1$.

Replacing (3) and (4) in (2) one obtains

$$\begin{aligned} &m_2\ddot{x}_2 = m_2 g - K_2(x_2 - x_1 - l_{20}) - C_2(\dot{x}_2 - \dot{x}_1), \\ &-C_2\dot{x}_1 - K_2 x_1 + m_2\ddot{x}_2 + C_2\dot{x}_2 + K_2 x_2 = m_2 g + K_2 l_{20}. \end{aligned} \tag{5}$$

The dynamical equation for the mass (1) is

$$m_1\ddot{x}_1 = m_1 g + F_{a_1} + F_{e_1} - F_{a_2} - F_{e_2} \,, \tag{6}$$

where x_1 is the coordinate, g − the gravitational acceleration, F_{e_1} − the elastic force and F_{a_1} − the damping force of the first spring.

The elastic force is given by Hooke's relation (Buzdugan *et al.*, 1979; Teodorescu, 1984; Teodorescu, 1988; Teodorescu, 1997):

$$F_{e_1} = -K_1(l_1 - l_{10}) = -K_1(x_1 - l_{10}) \,. \tag{7}$$

The damping force is given by (Buzdugan *et al.*, 1979; Teodorescu, 1984; Teodorescu, 1988; Teodorescu, 1997):

$$F_{a_1} = -C_1\dot{x}_1 \,. \tag{8}$$

Adding the eqs. (2) and (6) one obtains

$$m_1\ddot{x}_1 + m_2\ddot{x}_2 = (m_1 + m_2)g + F_{e1} + F_{a1} \,. \tag{9}$$

Replacing (7) and (8) in (9) one obtains

$$m_1\ddot{x}_1 + C_1\dot{x}_1 + K_1 x_1 + m_2\ddot{x}_2 = (m_1 + m_2)g + K_1 l_{10} \,.$$

and for the case $m_1 = 0$ it results

$$C_1\dot{x}_1 + K_1 x_1 + m_2\ddot{x}_2 = m_2 g + K_1 l_{10} \,. \tag{10}$$

The eqs. (5) and (10) gives the following system

$$\begin{cases} C_1\dot{x}_1 + K_1 x_1 + m_2\ddot{x}_2 = m_2 g + K_1 l_{10} \\ -C_2\dot{x}_1 - K_2 x_1 + m_2\ddot{x}_2 + C_2\dot{x}_2 + K_2 x_2 = m_2 g + K_2 l_{20} \end{cases} \tag{11}$$

3. Solution of the Dynamical Equation

This is a system of two linear differential equations in the unknowns x_1 and x_2. The system can be integrated with a supplementary hypothesis:

$$\frac{C_2}{C_1} = \frac{K_2}{K_1} = h \,, \tag{12}$$

where h is a positive constant.

Multiplying the first eq. (11) with h and adding the equations one obtains

$$m_2(h+1)\ddot{x}_2 + C_2\dot{x}_2 + K_2 x_2 = m_2(h+1)g + h\,k_1 l_{10} + k_2 l_{20} \,,$$

or also

$$m_2\frac{K_1 + K_2}{K_1}\ddot{x}_2 + C_2\dot{x}_2 + K_2 x_2 = \frac{K_1 + K_2}{K_1} m_2 g + K_2(l_{10} + l_{20}) \,. \tag{13}$$

The solution of this equation is

$$x_2 = e^{-\lambda t}(x_{21}\cos\omega\ t + x_{22}\sin\omega\ t) + \frac{K_1 + K_2}{K_1 K_2} m_2 g + l_{10} + l_{20} \,,$$

where λ and ω are given by (Buzdugan *et al.*, 1979; Teodorescu, 1984; Teodorescu, 1988; Teodorescu, 1997):

$$\lambda = \frac{C_2 K_1}{2m_2(K_1 + K_2)} = \frac{C_2}{2m_2(h+1)} \,. \tag{14}$$

$$\omega = \sqrt{\frac{K_1 K_2}{m_2(K_1 + K_2)} - \frac{C_2^2 K_1^2}{4m_2^2(K_1 + K_2)^2}}$$

$$\omega = \sqrt{\frac{K_2}{m_2(h+1)} - \frac{C_2^2}{4m_2^2(h+1)^2}}\ , \tag{15}$$

while x_{21} and x_{22} are constants of integration.

In the hypothesis of subcritical damping for the second spring the values of ω are real:

$$C_2^2 < 4K_2 m_2, \qquad C_2^2 < 4K_2 m_2(h+1),$$

$$\frac{K_2}{m_2(h+1)} > \frac{C_2^2}{4m_2^2(h+1)^2}\ .$$

For the determination of equivalent damping constant one writes from relation (14):

$$\lambda = \frac{C_2}{2m_2(h+1)} = \frac{C_{ech}}{2m_2}\ , \tag{16}$$

because the mass of the dynamical system is m_2.

Replacing h from (12) in (16) it results

$$C_{ech} = \frac{C_1 C_2}{C_1 + C_2}\ ,$$

or also

$$\frac{1}{C_{ech}} = \frac{1}{C_1} + \frac{1}{C_2}\ , \tag{17}$$

relation analogous with the equivalent elastic constant of series assembly.

The pulsation ω can be written into the form

$$\omega = \sqrt{\frac{K_{ech}}{m_2} - \frac{C_{ech}^2}{4m_2^2}}\ ,$$

where K_{ech} is the equivalent elastic constant (Buzdugan *et al*., 1979):

$$K_{ech} = \frac{K_1 K_2}{K_1 + K_2} . \qquad (18)$$

4. Conclusions

In conclusion, the dynamical study can be made replacing the series assembly with an equivalent spring having the characteristics (elastic constant and damping constant) given by relations (17) and (18).

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DETERMINAREA CONSTANTEI DE AMORTIZARE ECHIVALENTE PENTRU UN ANSAMBLU DE ARCURI LEGATE ÎN SERIE

(Rezumat)

Se consideră un sistem elastic compus din două arcuri legate în serie cu o masă ataşată în poziţie verticală. Ecuaţia dinamică a mişcării în regim de vibraţii libere se compară cu ecuaţia dinamică a sistemului echivalent cu un singur arc şi se determină constanta de amortizare echivalentă a montajului serie.